GENERATION OF LARGE PHOTON-NUMBER CAT STATES USING LINEAR OPTICS AND QUANTUM MEMORY*

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A recursive method for producing path-entangled states of light is presented. These states may find applications in quantum lithography and high-precision interferometric measurements. The required resources are single-photon sources, linear optics components, and photodetectors. Adding a quantum memory greatly enhances the yield in comparison with the previously known schemes.

Entangled states play a fundamental role in quantum information processing as they are basic ingredients in various tasks such as quantum teleportation, quantum key distribution, or quantum computing [1]. Entanglement has also been shown to be a useful resource for high-precision frequency measurements or quantum lithography [2,3]. In this paper, we will focus on the entangled states needed in the latter applications, that is, photon-number path-entangled states or *cat states*. These states are of the form

$$|N_+\rangle_{a,b} = \frac{1}{\sqrt{2N!}}(a^{*N} + b^{*N})|0\rangle, \tag{1}$$

where a^* and b^* denote the bosonic creation operators for two modes of the electromagnetic field, and $|0\rangle$ denotes the vacuum state. Deterministic schemes to produce cat states using $\chi^{(3)}$ nonlinearity have been proposed [4,5]. Unfortunately, the presently available $\chi^{(3)}$ couplings are about 10^{16} times weaker than the required values, which makes these schemes out of the scope of current technology. Recently, however, alternative schemes that only require photon-number state sources, linear optics components (beam splitters and phase shifters) and photodetectors have been proposed for $N = 4$ first[6], and then for an arbitrary photon number N[7,8,9]. These schemes are unfortunately *probabilistic*, and the probability $p(N_+)$ of a successful state preparation scales exponentially poorly with N for all of these schemes. Typically, if N is even, $p(N_+)$ scales as c^{-N} with the constant $c = \sqrt{2}\, e$ for the scheme [7] and $c = e$ in [8]. This is so because these schemes work by adding (or subtracting) the photons one by one (or two by two), each step having a non-unity probability of success. Such a multiplicative process (the yield is a product of success probabilities) naturally scales exponentially.

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In this paper, we will demonstrate that using a *tree-like* structure for the linear optics circuit makes it possible to significantly enhance the yield. We will describe a recursive scheme that generates a state of the form of Eq. (1) when N is a power of 2 with an approximate yield of

$$(4\sqrt{\pi})^{-\log_2 N} N^{(1-\log_2 N)/4}. \tag{2}$$

Thus, in contrast with the previous schemes, the scaling is *sub-exponential* in N, which exploits the fact that only $\log_2 N$ recurrence levels are needed in the tree. As we will see, the price to pay is that a quantum memory (or fast optical switches) is required in addition to the other optical components.

The basic operation in our method is the transformation T_N, which probabilistically transforms two cat states of N photons into one cat state containing $2N$ photons, that is, $T_N : |N_+\rangle_{a,b} \otimes |N_+\rangle_{c,d} \to |2N_+\rangle_{a,b}$ with probability $p(T_N)$. The transformation T_N is realized as follows. First, change the phase of mode d as $d \to e^{i\pi/N} d$. Next, combine the modes a and c at a 50:50 beam splitter and modes b and d at another 50:50 beam splitter. Defining the action of a 50:50 beam splitter on two impinging modes a and b as $a \to (a - ib)/\sqrt{2}$ and $b \to (-ia + b)/\sqrt{2}$, we obtain the 4-mode state

$$\frac{1}{2^{N+1} N!} \left((a^* + ic^*)^N + (b^* + id^*)^N \right) \left((ia^* + c^*)^N - (ib^* + d^*)^N \right) |0\rangle. \tag{3}$$

Finally, measure the number of photons contained in modes c and d. The transformation succeeds if *no* photon is detected in these two modes, in which case we get the (unnormalized) state

$$\frac{1}{2^{N+1} N!} \left((a^*)^{2N} - (b^*)^{2N} \right) |0\rangle. \tag{4}$$

Transforming the phase of mode b as $b \to e^{i\pi/2N} b$ then yields the desired state $|2N_+\rangle_{a,b}$ with a success probability $p(T_N) = \frac{(2N)!}{2^{2N+1}(N!)^2}$. Upon using the Stirling formula, we get

$$p(T_N) \simeq \frac{1}{\sqrt{4\pi N}} \left(1 + O(1/N)\right). \tag{5}$$

Remarkably, $p(T_N)$ does not decrease exponentially with N as if the photons had been added one by one, but only as $1/\sqrt{N}$, so the successive steps of our iterative process keep a reasonably good efficiency as N increases.

Consider first a naive scheme that does not use a quantum memory. Starting from N one-photon states $|1_+\rangle$, which are easily obtainable with a single-photon source and a beam splitter, one probabilistically produces $N/2$ states $|2_+\rangle$, then $N/4$ states $|4_+\rangle$, and so on until one gets the desired state $|N_+\rangle$. What is the overall probability $p(N_+)$ to produce $|N_+\rangle$ with this procedure? Solving the recurrence equation $p(2N_+) = p(N_+)^2 p(T_N)$, we get

$$p(N_+) = \frac{2N!}{(2N)^N} \simeq \sqrt{8\pi N}\, (2e)^{-N} \left(1 + O(1/N)\right). \tag{6}$$

The reason why this success probability decreases exponentially with N is that we require all the $(N-1)$ individual transformations T at all recurrence levels to succeed simultaneously. Since each transformation T has a non-unity probability of success, the probability that they all succeed is exponentially small.

Let us now show that we can strongly improve on this by using a quantum memory. We start with $|1_+\rangle^{\otimes M_1}$ where the number M_1 of states we need to prepare initially will be determined below. Suppose that, after a few steps, we have obtained the state $|n_+\rangle^{\otimes M_n}$, assuming M_n is an even number. We carry out the transformation T_n on each pair of states, which produces on average M_{2n} states $|2n_+\rangle$ where $M_{2n} = \frac{M_n}{2} P(T_n)$. We then discard the states for which the transformation failed and only keep those for which it succeeded (there are, on average, M_{2n} such states). We use these states to construct the state $|4n_+\rangle$, and so on. If we want to produce M_N cat states $|N_+\rangle$ (where N is a power of 2) with a probability of order one, then, solving the above recurrence using Eq. (5), we find that the average number M_1 of states $|1_+\rangle$ that are needed initially is

$$M_1 \simeq (4\sqrt{\pi})^{\log_2 N} N^{(\log_2 N - 1)/4} \left(M_N + \mathcal{O}(\sqrt{M_N}) \right), \tag{7}$$

which immediately implies Eq. (2). This shows that the resources (*i.e.*, the number of single photons, of operations T_n, of beam splitters, of photodetections, etc.) needed for generating a cat state increase only sub-exponentially with N. This recursive scheme thus requires much less operations than the naive scheme, but the dynamics is now conditional on whether the operations T_n succeed or not. Such a conditional dynamics requires fast optical switches[10] in order to store the photons in a loop, or, alternatively, a quantum memory[11] in which the photon states can be stored. These extra technological resources make our scheme much more demanding than those based only on linear optics, single photon sources, and photodetectors only [6,7,8,9], but $\chi^{(3)}$ couplings are still not required. Note that we may start our recursion from two-photon cat states $|2_+\rangle$ (instead of $|1_+\rangle$) since they can be deterministically generated from two one-photon states using a beam splitter[12], resulting in an extra factor of 4 in the yield Eq. (2).

Our scheme is sensitive to various imperfections such as the losses in the beam splitters, imperfect mode-matching, etc. Let us focus on one of them, namely the non-unity detector efficiency $\eta < 1$. The probability that the detector does not click when it has absorbed j photons is given by $(1-\eta)^j$. At first sight, it seems that the main source of errors will be due to the events where no photon is detected in modes c and d while a photon has actually been absorbed in either one of the detectors. By expanding Eq. (3) in a series in c^* and d^*, we find that the probability that there is a single photon in mode c or d is $p(c^*c + d^*d = 1) = \frac{2N}{2^{2N}}$. This probability is thus exponentially smaller than $p(T_N)$, so this event can in fact be neglected in good approximation. Next, if we calculate the probability that two photons are in mode c or d, $p(c^*c+d^*d=2)=\frac{(2N-2)!}{2^{2N+1}(N-1)!^2}\simeq\frac{P(T_N)}{4}$, we find that, for large N, the probability that one decides that T_N succeeded while it actually failed is approximately given by $(1-\eta)^2/4$. So we conclude that our scheme is robust with respect to detector inefficiencies up to the second order in $(1-\eta)$.

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